

Q1 Nyquist's Criterion

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(a) In the time domain :

$$p(t) = \begin{cases} 1, \\ 0, \end{cases}$$

$$t = 0$$

$$t = \pm T, \pm 2T, \pm 3T, \dots$$

↑ symbol "duration"
"interval"

→ T does not necessarily equal the pulse duration (the pulse may not even be time-limited.)

$$\frac{1}{T} = \text{signaling rate}$$

measured in symbols per second or baud

(b) In the freq. domain :

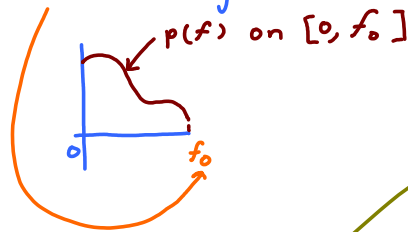
$$\star \sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T$$

Reminder: A pulse $p(t)$ is called a Nyquist pulse iff it satisfies \star

Q2 Nyquist Pulses

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General strategy (recipe): Let the end of the given interval be $\frac{1}{2T}$.



These are chosen to ease the design.

Then, set the symbol duration to be $T = \frac{1}{2f_0}$.

Our pulse will be band-limited to $\frac{1}{T}$.

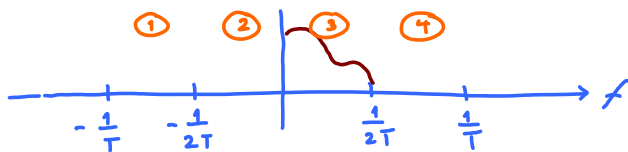
(Will return to explain them later.)

Recall that to check whether a pulse is a Nyquist pulse,

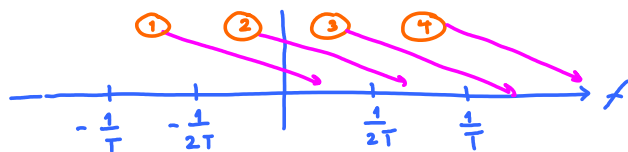
$$\sum_{k=-\infty}^{\infty} P(f - \frac{k}{T}) \equiv T.$$

$P(f)$ is replicated every $\frac{1}{T}$.

Now consider 4 intervals:



Note that when $P(f)$ is copied to $\frac{1}{T}$, its content in region ① will show up in region ③.



Therefore, when we add $P(f)$ in region ① and $P(f)$ in region ③, we must have T .

In other words, $P(f)$ in region ① can be found by $T - P(f)$

in region ③.

(hinted)

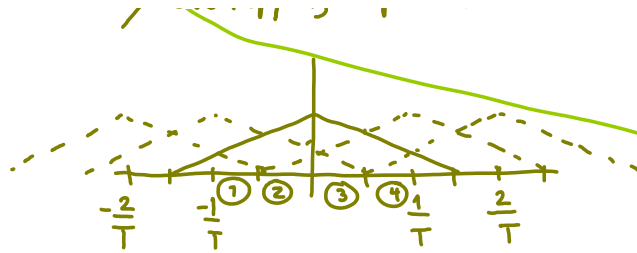
The suggested symmetry in $P(f)$ allow us to find

$P(f)$ in region ② by flipping $P(f)$ in region ③ horizontally

and

$P(f)$ in region ④ by flipping $P(f)$ in region ① horizontally.

Remark: We don't want $P(f)$ to be non-zero outside region ①-④ because when we apply $\sum_{k=-\infty}^{\infty} P(f - \frac{k}{T})$, if $P(f)$ is too wide, we will have to deal with many overlapping replicas.



We don't want $P(f)$ to be zero inside region (4) (and hence (1)) because we may need it to "cancel" the region (3) value to get the Σ to be T .

This is why we set the provided portion of $P(f)$ to be in region (3).

For this question, we have $f_0 = 1$. Therefore, we will choose

$$T = \frac{1}{2f_0} = 0.5 \Rightarrow \frac{1}{T} = 2 \text{ and } \frac{1}{2T} = 1.$$

The four regions under consideration are

- (1) $f \in [-2, -1)$
- (2) $f \in [-1, 0)$
- (3) $f \in [0, 1)$
- (4) $f \in [1, 2]$

The question specifies $P(f)$ in region (3) as discussed above.

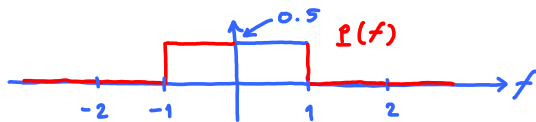
We will use the strategy described above to find $P(f)$ which is band-limited to $\frac{1}{T}$.

(a)



Note that in region (3), $P(f) \equiv T$ already. Therefore, we need nothing from region (1); in other words, $P(f) \equiv 0$ in region (1).

The values in region (2) and (4) are from the symmetry in $P(f)$.



(There is no need to check whether $P(f)$ satisfies the Nyquist criterion again because it is constructed by our recipe above.)

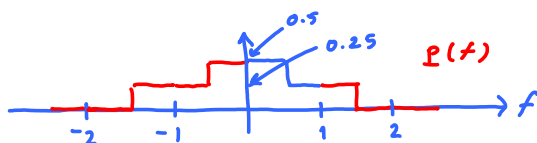
(b) In this part, the value of $P(f)$ in region (3) is 0.25;

so, we will set the value in region (1) to be $T - 0.25 = 0.5 - 0.25 = 0.25$.

The values in region (2) and (4) are from the symmetry in $P(f)$.



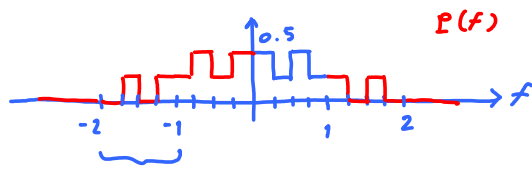
(c)



As before, the value in region (1) is $T - \text{region (3)} = 0.5 - \int_0^{0.5} 0.25 = 0.25$.

As before, the value in region ① is T-region ③ = 0.5 - $\int_{-2}^{-1} f(f) df$.

(d)



$$\int_{-2}^{-1} f(f) df = 0.5 - \int_{-2}^{-1} f(f) df = \int_{-1}^0 f(f) df$$

Q3 Raised Cosine Pulse

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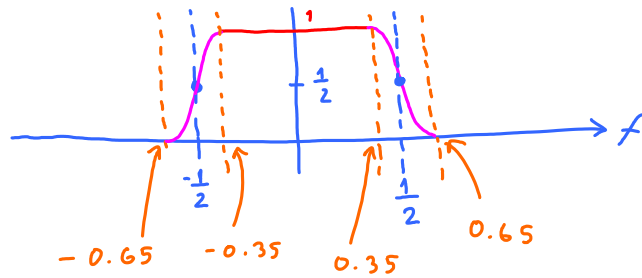
$$\alpha = 0.3, T = 1$$

(a)

$$\frac{1}{2T} = \frac{1}{2}$$

$$\frac{\alpha}{T} = \frac{0.3}{1} = 0.3$$

$$\frac{\alpha}{2T} = \frac{0.3}{2} = 0.15$$



(b) The raised cosine pulse is a Nyquist pulse.

Therefore,

$$p(t) = \begin{cases} 1, & t=0 \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

Here, $T = 1$.

Hence,

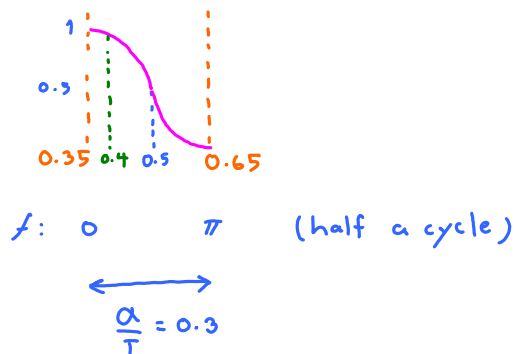
$$p(t) = \begin{cases} 1, & t=0 \\ 0, & t = \pm 1, \pm 2, \pm 3, \dots \end{cases}$$

In particular, $p(2) = 0$

(c) Note that $P(\frac{1}{2T}) = \frac{1}{2}$. Here, $T = 1$. Hence, $P(\frac{1}{2}) = \frac{1}{2}$.

(d) From part (a), we've seen that $P(f) = 1$ for $f \in [-0.35, 0.35]$. Therefore, $P(0.3) = 1$.

(e) We magnify the plot in part (a) for clarity



This is the "raised" part of RC.

$$\Rightarrow P(0.4) = \frac{1}{2} \left(1 + \cos \left(\pi \times \frac{0.05}{0.3} \right) \right)$$

$$= \frac{1}{2} \left(1 + \cos \left(\frac{\pi}{6} \right) \right)$$

$$= \frac{2 + \sqrt{3}}{4} \approx 0.933$$

Alternatively, although not recommended, we can find the value of $P(0.4)$ simply by plugging in the formula for the raised cosine given in lecture.

The value of P in the cosine part is given by $\frac{1}{2} \left(1 + \cos \left(\frac{\pi T}{\alpha} \left(|f| - \frac{1-\alpha}{2T} \right) \right) \right)$.

In this part, $f = 0.4$, $\alpha = 0.3$, and $T = 1$.

Hence,

$$P(0.4) = \frac{1}{2} \left(1 + \cos \left(\frac{\pi \times 1}{0.3} \left(0.4 - \frac{0.7}{2} \right) \right) \right) = \frac{1}{2} \left(1 + \cos \left(\frac{\pi}{6} \right) \right) \leftarrow \text{same as above.}$$

0.05

Note: $\cos(\frac{\pi}{6}) \approx 0.866$

↑ this is in radians (not degrees)

If you plug $\frac{\pi}{6}$ into the cos function when your calculator is in deg mode, you would get the incorrect number $\cos(\frac{\pi}{6}^\circ) \approx 1$

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(0.999958244)

Q4 Raised Cosine Pulse

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(slide)

We have seen in class that the formula for the raised cosine pulse in the time domain is

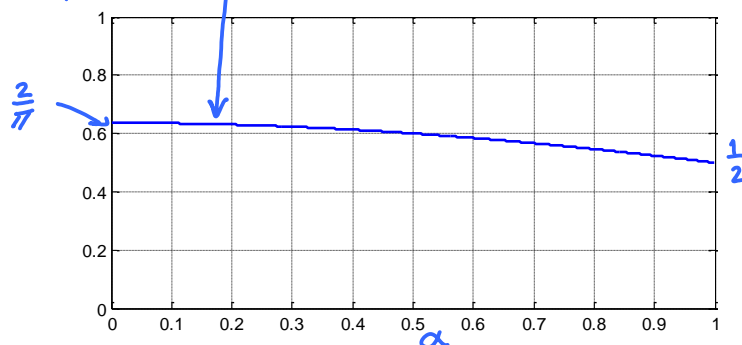
$$p_{RC}(t; \alpha) = \frac{\cos(\alpha \pi \frac{t}{T})}{1 - (2\alpha \frac{t}{T})^2} \frac{\sin(\pi \frac{t}{T})}{\pi \frac{t}{T}}$$

At $t = \frac{T}{2}$, we have $\frac{t}{T} = \frac{1}{2}$

$$p_{RC}\left(\frac{T}{2}; \alpha\right) = \frac{\cos\left(\alpha \frac{\pi}{2}\right)}{1 - \alpha^2} \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}}$$

$$= \frac{2}{\pi} \times \frac{\cos\left(\alpha \frac{\pi}{2}\right)}{1 - \alpha^2}$$

MATLAB plot:



Note that when $\alpha = 0$, $p_{RC}\left(\frac{T}{2}; \alpha\right) = \frac{2}{\pi} \approx 0.6366$

As $\alpha \rightarrow 1$, $p_{RC}\left(\frac{T}{2}; \alpha\right) \rightarrow \frac{2}{\pi} \times \frac{\frac{\pi}{2}(-\sin(\alpha \frac{\pi}{2}))}{-2\alpha} \Big|_{\alpha=1} = \frac{1}{2}$.